Growth at Risk: methodology and applications in an open-source platform*

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Abstract

This article describes the construction of an open-source growth-at-risk (GaR) model. The model provides a flexible analytical tool for policymakers and researchers aiming at using the GaR approach to characterize the probability density of GDP growth conditional on information contained in domestic and international macrofinancial variables. This article, together with its related online repository, aims at fostering the understanding of macrofinancial risk factors both in advanced and emerging economies.

Keywords: financial stability, growth-at-risk, quantile regressions, economic growth.

JEL Codes: G01, O4, O47, E44.

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1 Introduction

This paper describes the application of a Growth-at-Risk (henceforth, GaR) model that provides an open-source baseline procedure that can be used by researchers and policymakers to work on different applications of this methodology. The paper provides a description of the GaR methodology and the main components of its related code. This description includes an ad hoc constructed Financial Conditions Index used to feed the GaR model. To facilitate future applications, the code is based on R as its programming language, allowing researchers to replicate the model using a free software environment.[1]

The GaR concept was originally developed as an extension of value-at-risk (VaR) models. While VaR models estimate expected investment losses conditional on market conditions, GaR models extend this idea to a macro level by estimating the expected distribution of GDP growth conditional on financial market conditions and other macroeconomic factors (henceforth, macrofinancial conditions). By connecting financial market conditions with expected GDP growth, GaR models provide an assessment of the real-sector implications of the build-up of systemic risk. As discussed below, GaR models can be also adjusted to be used as a platform for policy evaluation, providing signals on how the relationship between financial stress and real-sector outcomes vary, for instance, when macroprudential policies are in place.

GaR applications are characterized by two key parameters that define the scope of the analysis. First, a researcher needs to inspect the 'historical' relationship between macrofinancial conditions and GDP growth for one or several countries of interest. Most applications focus on periods between 20 to 30 years analyzed using data at a quarterly or monthly frequency. Second, a decision needs to be made regarding the time-horizon that will describe the relationship between macrofinancial conditions and future GDP growth. Typically, an analyst will be interested in

[1] The related material is available at the C-GARP website of the Center for Latin American Monetary Studies (CEMLA).

exploring the effect of macrofinancial conditions on GDP growth one or two quarters ahead.

Once the relationship between macrofinancial conditions and GDP growth has been established by using quantile regression methods, one can estimate the shape of the expected GDP growth distribution. Conditional on the stance of relevant macrofinancial conditions, this estimated distribution tells us that GDP growth is not expected to fall below a certain threshold with a high probability such as 95%, which defines the confidence level of the assessment. In other words, we can draw from the distribution an assessment of how 'low' the GDP growth rate could fall, provided that GDP growth falls, for instance, below the 5th percentile of its expected distribution.

Figure 1 provides a hypothetical example of a GaR estimation. It shows an estimated probability density function of GDP growth with a mean of 2.5%. An adverse scenario is defined, as a reference, as a realization of a GDP growth rate of 0%. In this example, the GaR with a probability of 5% is estimated at a GDP growth rate of -2.3%. These results means that with a probability of 5%, the GDP growth rate will be equal or lower than -2.3% h periods ahead.

Figure 1 represents a benchmark estimation that can be used to evaluate how the GaR value changes depending on the stance of macrofinancial conditions. An example of a counterfactual exercise of this type is depicted in Figure 2. This exercise begins by assuming a negative shock to macrofinancial conditions equal to a one standard deviation shift in a macrofinancial conditions indicator. As depicted, this shock shifts the entire expected density of GDP growth to the left (represented by the dashed red line). The area marked by the points A, B, and C reflects the impact of the negative shock on the left tail of the GDP growth distribution: it shows that the shock increases the probability of realizing a negative GDP growth rate proportionally to the size of this area.

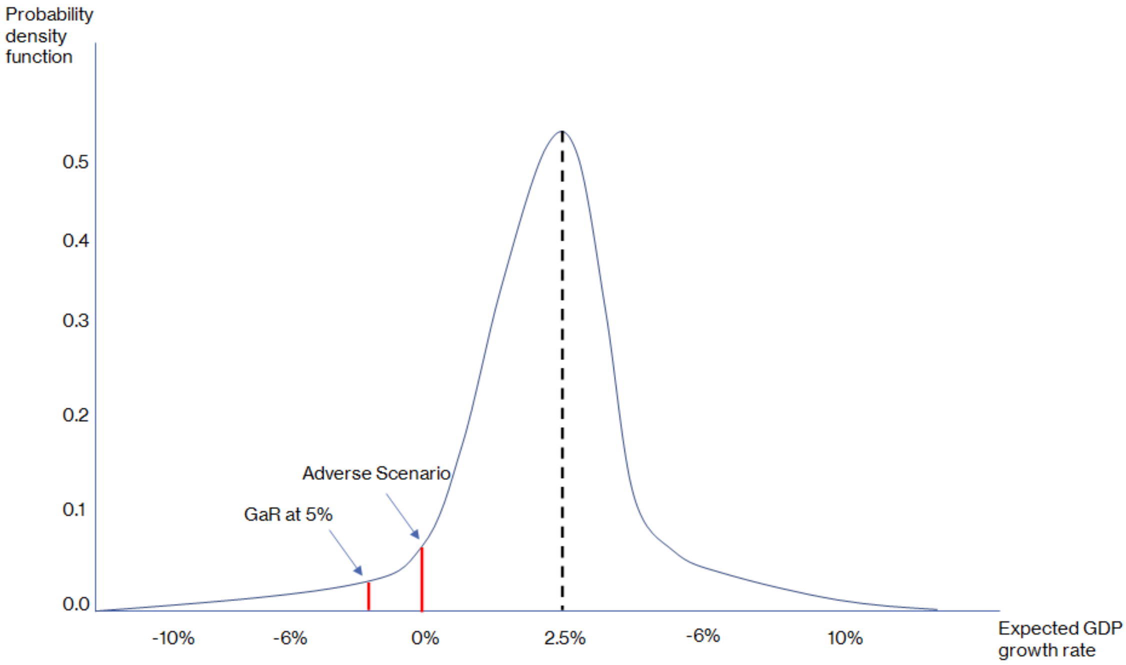


Figure 1: Example of a GaR Estimation. This graph describes an hypothetical example of a GaR estimation. The horizontal axis represents the estimated GDP growth rate along the expected distribution of GDP growth conditional on a given stance of macrofinancial conditions. A 'Stress Scenario' is represented by a GDP growth rate below 0%. GaR at 5% represents the maximum expected GDP growth rate (in this example -2.3%) that would be realized if GDP growth falls below the 5th percentile of its expected distribution. This scenario is expected to materialize with a probability of 5%.

This example highlights the advantages of the GaR method. Using a simple and parsimonious reduced-from forecasting system, an analyst can obtain estimates of how the expected GDP growth distribution changes when financial shocks hit an economy. Moreover, the method allows an analyst to select macrofinancial variables that better explain GDP growth trends, tailoring the model for different macroeconomic and institutional environments.

Despite these salient features, important caveats should be kept in mind when estimating GaR models. First, the lack of an identification structure when inspecting the relationship between GDP growth and macrofinancial conditions means that the estimation should not be interpreted as reflecting causal links. Second, the estimation provides a 'guess' about the expected density of GDP growth obtained

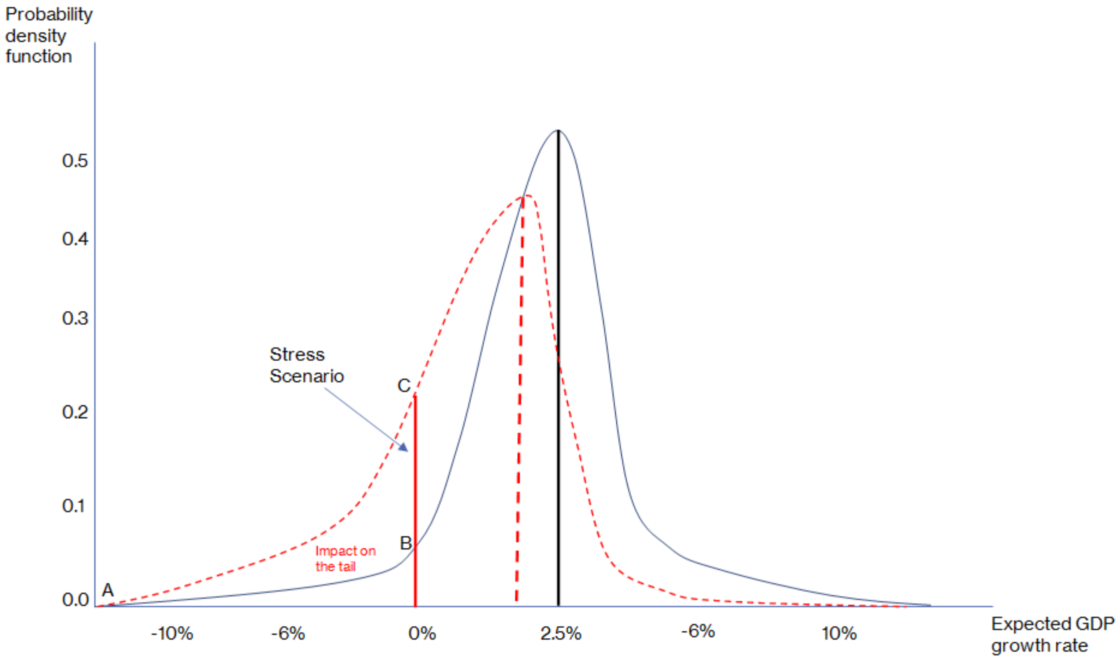


Figure 2: Example of a GaR counterfactual analysis. This graph describes a counterfactual analysis that can be performed with the GaR method. The horizontal axis represents the estimated GDP growth rate along the expected distribution of GDP growth conditional on a given stance of macrofinancial conditions. A 'Stress Scenario' is represented by a GDP growth rate of 0%. The dashed red curve represents a left-shift in the expected GDP growth distribution following a one standard deviation negative shock on macrofinancial conditions. The left-shift in the GDP growth distribution increases the probability of realizing a negative GDP growth rate by an amount equal to the A B C area ('impact on the tail').

through the lens of past events. To the extent that sunspot shocks in the past can bias the relationship between macrofinancial conditions and GDP growth, an analyst should be cautious when interpreting the findings.[2] Third, any policy advice derived from GaR models is limited given the lack of insights about underlying institutional mechanisms explaining the relationship between macrofinancial conditions and GDP growth. Therefore, GaR models work best when complemented with robustness checks and qualitative evidence that can shed light on the reasons explaining the phenomena identified in the estimations.

The applications of GaR models originate in seminal contributions by Giglio et al.

[2] In Section 4 we provide a discussion on how the COVID-19 pandemic illustrates the limitation of GaR models when assessing stress situations driven by exogenous and unexpected shocks not captured in economic fundamentals.

(2016) and Adrian et al. (2019). A complementary description of GaR methods focused on the use of GaR models at the International Monetary Fund (IMF) is provided by Prasad et al. (2019). Recently, the GaR method had been extended to be applied for policy evaluation in different settings. For example, Sánchez and Röhn (2016) show for a group of OECD countries that institutional factors such as a stronger banking supervision and larger international reserves are associated with more moderated shifts in expected GDP growth distributions when financial shocks hit. Aikman et al. (2019), Franta and Gambacorta (2020), and Galán (2020) provide evidence that active macroprudential policies mitigate the downside risks of low GDP growth in stress scenarios. Finally, the literature has been evolving to apply the GaR approach by looking at different outcome variables, such as capital flows (see, e.g., Gelos et al., 2019 or Eguren-Martin et al., 2021).

This project extends previous platforms to estimate GaR models, which include a Python-based framework developed at the IMF. Our approach is different in three ways. First, the programming code allows for more degrees of methodological flexibility as explained below, what matters when it comes to adapt a forecasting model to different institutional environments. Second, the model includes the calculation of a Financial Conditions Index that can be used as a monitoring tool, even beyond the GaR framework. Finally, the model was adjusted to allow using panel data estimations for multiple countries or regions within the GaR framework.

The rest of the paper is organized as follows. Section 2 describes the steps needed to implement and interpret a GaR estimation using an example from a panel of Latin American economies. Section 3 explains the construction of the ad hoc Financial Conditions Index used in the GaR estimation. Section 4 provides examples of how the GaR methodology can be extended for purposes of policy evaluation. Section 5 concludes.[3]

[3] In Section A.2 in the Appendix we further provide the definition and sources of the variables used in the GaR model.

2 Growth at Risk Methodology

2.1 Identifying risk factors

GaR applications begin with the identification of relevant macrofinancial variables that could arguably explain dynamics in GDP growth in a given country (see, e.g., Adrian et al., 2019 or Prasad et al., 2019). Researchers typically rely on a combination of local macroeconomic conditions, local macrofinancial conditions, and international macrofinancial conditions. The selection of variables will depend, for example, on a country's risk profile, its economic and financial openness, or the characteristics of its financial system.

Table 1 reports a diagram describing a few examples of relevant variables that have been used in GaR applications. The bucket of Local Macroeconomic Conditions includes broad proxies for the stance of the macroeconomic cycle. Most studies rely only on a lagged measure of GDP growth to capture trend effects. Measures of price and sovereign risk stability could also be considered for this purpose.

The second bucket shifts the focus towards macrofinancial conditions that are specific to a given country. Most studies look at proxies capturing trends in the credit market (e.g., Credit-to-GDP ratio), interest rate differentials vis-à-vis an international benchmark, and measures of exchange rate trends. Several applications rely on aggregated indices combining multiple dimensions of local macrofinancial conditions. We present an example of an index in Section 3.

Finally, international macro conditions include measures of global financial stress that capture common risk factors across countries beyond the idiosyncratic characteristics of the analyzed economies. Studies often rely on measures of global stock market volatility such as the Chicago Board Options Exchange Market Volatility Index (VIX Index), measures of capital flows volatility, or global US Dollar price indices. To provide a clear statistical distinction between the three buckets, re-

GDP Risk Factors		
Local Macroeconomic Conditions	**Local Macrofinancial Conditions**	**International Macro Conditions**
Past GDP growth	Credit-GDP ratio	Stock market index
Sovereign Risk	Interest rates	Capital flows volatility
Inflation	Exchange rates	US Dollar Index

Table 1: Selection of GDP risk factors. This diagram shows a categorization of relevant GDP risk factors often used in the GaR literature. Most studies consider three buckets of variables that could potentially explain GDP growth prospects: Local Macroeconomic Conditions, Local Macrofinancial Conditions, and International Macroeconomic Conditions. The diagram lists a few examples that could be used within each bucket. It should be noted that the selection of variables depends first on a qualitative assessment of relevant factors that impact GDP growth in a specific country or group of countries.

searchers typically orthogonalize the international conditions bucket with respect to local macrofinancial conditions. This adjustment can be done, for instance, by using only the residuals of a regression of international against local conditions as a proxy for global risk factors.

Three aspects related to the selection of GDP risk factors should be remarked. First, most GaR applications rely on a reduced number of variables. Second, if a bucket considers more than one variable, researchers often opt for grouping these variables into a single indicator. Finally, researchers often choose to consider some variables measured both in levels and in terms of their volatility as separate variables entering a GaR model. This feature is also explained in the context of the Financial Conditions Index described in Section 3.

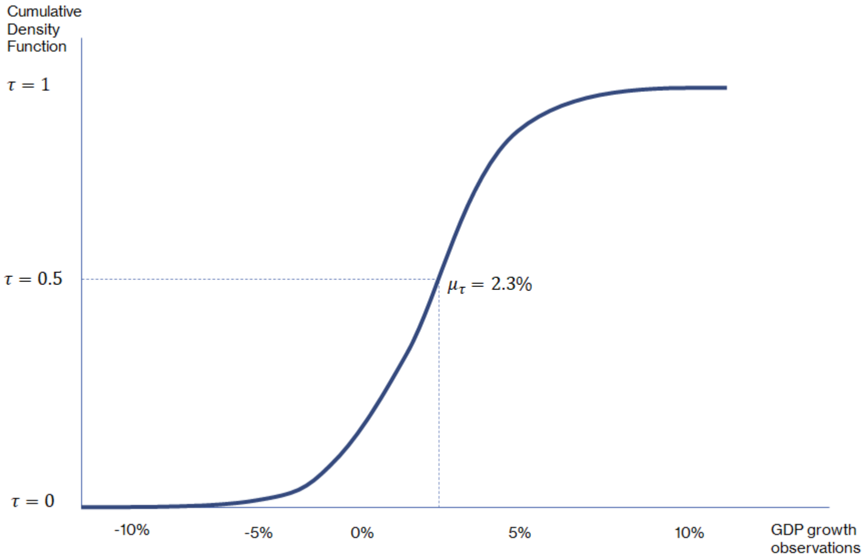


Figure 3: Example of a Cumulative Distribution Function (CDF). This figure depicts an hypothetical cumulative distribution function of GDP growth. The x-axis represents the size-ranked GDP growth observations. The y-axis shows the accumulated share of observations covered below (i.e. to the left) of each GDP growth observations. For example, the 0.5 quantile (or 50th percentile) of the distribution at $\tau = 0.5$ represents a GDP growth rate of 2.5%. Therefore, GDP growth is expected to be equal or smaller than 2.5% with a probability of 50%.

2.2 Quantile regressions

Having identified relevant macrofinancial risk factors, the next step is to estimate the relationship between these risk factors and GDP growth using quantile regressions. This method allows characterizing the entire probability density of GDP growth conditional on the stance of macrofinancial conditions. Therefore, we are not interested in point estimates of GDP growth but rather in understanding how macrofinancial conditions affect the distribution of expected GDP growth. Quantile regressions explore heterogeneous responses along the distribution of GDP growth asking, for instance, whether the left tail of the distribution experiences a greater sensibility to macrofinancial conditions.

Quantile regressions can be described as a generalization of OLS estimations in which we estimate the expected mean of a dependent variable conditional on a set of covariates:

$$\mathbb{E}(y|x) = x\beta_m \tag{1}$$

In Eq. 1, the estimated effect is represented by the term β_m, which describes the sensitivity of the mean dependent variable to changes in the covariate x. The quantile regression approach is different in that we estimate the effect of x on a given percentile of the GDP growth distribution.[4] That is, the regression will result in as many coefficients as quantiles τ are being considered for the analysis.

$$Percentile_{\tau}(y|x) = x\beta_{\tau} \tag{2}$$

Eq. 2 highlights that the interpretation of quantile regressions is about changes in the GDP growth rate that represent a given percentile of its distribution conditional on covariates x. Formally, the τ quantile of the variable y represents a threshold value of y such that with a probability of τ, y will be equal or smaller than this threshold. We label this quantile μ_{τ} as follows:

$$\tau = P(GDP \leqslant \mu_{\tau}) \equiv F_{GDP}(\mu_{\tau}) \tag{3}$$

Eq. 3 shows that the τ quantile of GDP growth can be drawn from the respective Cumulative Density Function (CDF) of GDP growth (F_{GDP}), which connects the observed GDP growth rates with the accumulated probability over the distribution. An example of a CDF function is reported in Figure 3, which depicts the distribution of observed values for GDP growth (x-axis) against the percentiles of the distribution, labeled as τ. In this example, the 0.5 quantile of the distribution

[4] Note that quantiles are defined as points along a distribution that refer to the rank order of values. Percentiles can be understood as a description of quantiles relative to 100. For example, the quantile 0.5 represents the 50th percentile of a distribution.

equals $\mu_\tau = 2.5\%$. Therefore, we expect GDP growth to be equal or smaller than 2.5% with a probability of 50%.

In a quantile regression we are interested in estimated quantiles that emerge conditional on the stance of explanatory variables. The intuition behind this approach is represented in Figure 4. Panel A shows a scatter plot of GDP growth (y-axis) against a financial conditions index ranging from 0 to 1, where 1 implies tighter financial conditions (FCI, x-axis). Panel B depicts the distribution of GDP growth observations for a given value range of FCI around its median. This panel also depicts red marks representing the quantiles of GDP growth for each value of FCI. Panel C shows these quantiles for three values of FCI in the data.

The quantile regression is finally depicted in Panel D, which contrasts the estimated linear regressions for the 0.2th (red line) and 0.8th (blue line) quantiles of GDP growth. In this example, a one percentage point (p.p.) increase in FCI leads to a 0.6 p.p. decrease in the 0.2th quantile of the GDP growth distribution. Therefore, we would expect that increases in FCI will shift the left-tale of the GDP growth distribution further to the left. On the contrary, FCI cannot significantly explain the 0.8th quantile of GDP growth. In this example, an increase in FCI leads therefore to larger probabilities of negative growth, while it does not affect the probability of higher GDP growth rates.

The exercise depicted in Figure 4 highlights the benefits of the quantile regression approach in GaR models: it allows a researcher to explore how changes in macrofinancial conditions deferentially affect the quantiles of the GDP growth distribution. Formally, a least square estimation solves a model for the parameter estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ by taking those values of the parameters that minimize the sum of squared residuals. The intuition of this approach is that the estimated function will approach

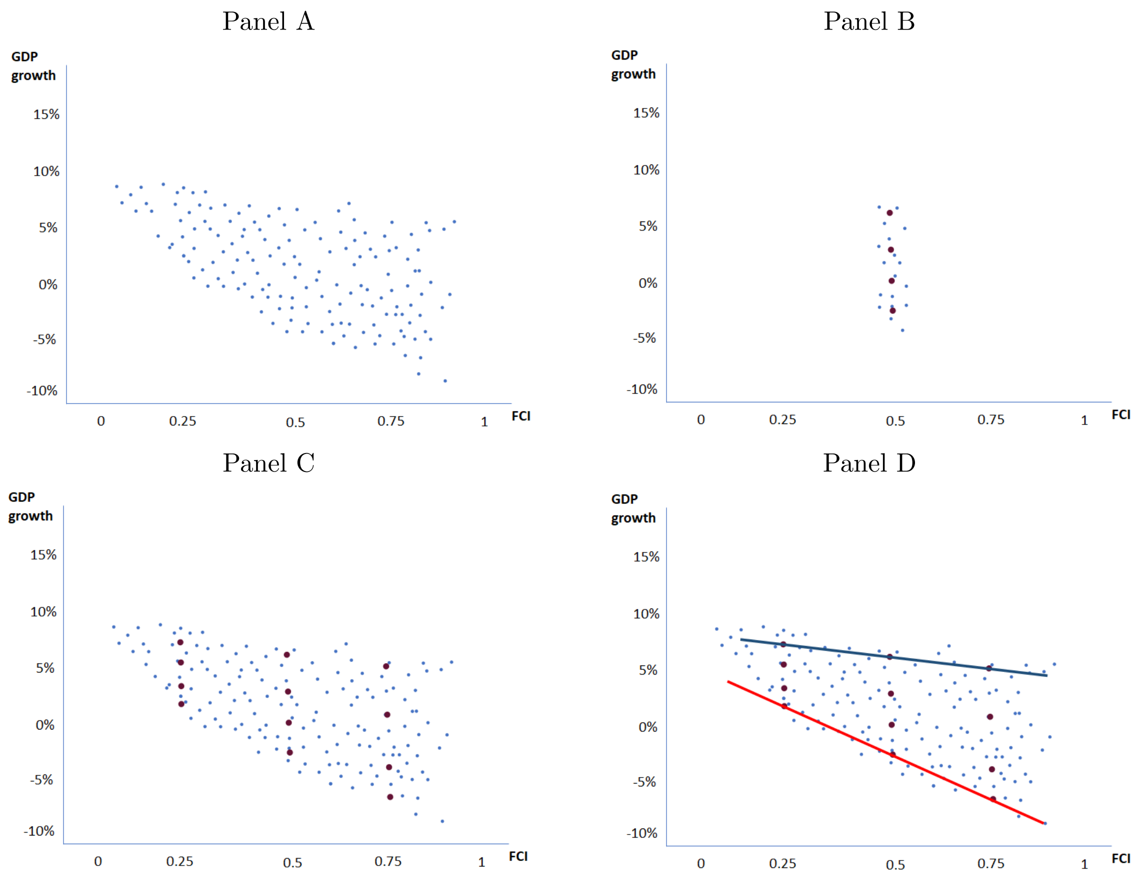


Figure 4: Intuition of a quantile regression. This panel describes the intuition behind a quantile regression estimation. Panel A depicts the (hypothetical) relationship between GDP growth (y axis) and a financial conditions index (FCI, x axis) for a set of countries in a given quarter. Panel B restricts the sample to GDP-growth observations around a value of 0.5 in the FCI. In this panel, the marks represented by red circles show the quantiles of the GDP growth distribution evaluated at different values of the FCI. Panel C replicates this latter exercise for three values of FCI. Panel D further adds two lines representing linear regressions at the 80th (red line) and 20th (blue line) percentiles of GDP growth.

the population's conditional mean for large samples:

$$min \sum_i (y_i - (\beta_0 + \beta_1 x_1))^2 \tag{4}$$

The quantile regression (QR) approach can be seen as a generalization of this linear regression method. Recall that a sample median (i.e., the quantile 0.5) can be computed as the value m of a distribution that minimizes the mean absolute distance between Y and m (see, e.g., Hao and Naiman, 2007, for details). We can label this average distance as $E\,|Y - m|$. By construction, the minimum distance $E\,|Y - m|$ is reached when the estimated median m has the same number of data points left and right from its estimated value. Similarly, the quantile $q = 0.8$ (i.e., the 80th percentile) of a distribution can be understood as a minimization of the sum of weighted distances from the sample points. The weights are represented by the value q for data points above the estimated quantile and $1 - q$ for data points below the estimated quantile. This idea is represented in Eq. 5:

$$Q(\beta_q) = \sum_{i:y_i \geq x_i'\beta}^{N} q \left| y_i - x_i'\beta_q \right| + \sum_{i:y_i < x_i'\beta}^{N} (1-q) \left| y_i - x_i'\beta_q \right| \tag{5}$$

Eq. 5 represents the empirical equation of a QR model. It shows that we seek to estimate a function of estimated quantiles (β_q) represented by the minimum weighted sum of distances between a data point and its estimated quantile. The weights q and $1-q$ ensure that the quantiles will be estimated such that there is a share q of data points below each quantile. For example, the function $Q(\beta_{0.8})$ would be estimated such that coefficient $\beta_{0.8}$ represents the quantile 0.8 of a distribution with 80% of the respective data points below the estimated quantile.

This approach has three key features that facilitate the econometric analysis compared to a linear regression model. First, a QR model has a monotone equivariance property, meaning that any monotone transformation of the variable y (e.g., taking its log) will result in estimating a quantile q subjected to the same transformation. Second, QR models do not require the usual homoscedasticity assumption from linear regression models. This assumption states that the error terms in the model are constant over the distribution of an explanatory variable x. Third, QR models are insensitive to outliers, in opposite to estimations of conditional means.

Standard errors are typically obtained in QR models via bootstrap methods. The usual analytical asymptotic standard errors face the problem of strongly relying on the assumption that the error terms are independent and identically distributed. Given that this assumption rarely holds, researchers often estimate standard errors using bootstrap methods. This approach relies on Monte-Carlo simulations to estimate a 'sample' distribution of a parameter (i.e., a standard error in our case) by taking multiple random samples from the data. In practice, one estimates the function $Q(\beta_q)$ multiple times in order to get a standard deviation of its slope, which can then be used for statistical inference. We refer to Hao and Naiman (2007) for details on computing confidence intervals in QR models.

From the perspective of econometric identification, a few estimation conditions need to be carefully addressed in the GaR context. First, the econometric QR model will typically consider lags between the dependent variable and the macro-financial variables. The number of lags with which the explanatory variables enter the model will define the forecasting horizon once the expected GDP distribution is constructed. Second, a researcher needs to access, if possible, quarterly or monthly macroeconomic data for long periods (typically 10 years at least) in order to maximize the observations and obtain a proper identification.

Third, if several countries or regions are being considered in a panel model, the definition of the variables must be carefully revised. Panel models have the advantage of allowing the inclusion of country fixed effects, mitigating concerns about omitted variable bias. In these cases, Eq. 5 must be adjusted, with the estimated quantiles $\hat{Q}$ taking the following form:

$$\hat{Q}^{\tau}_{i,t}(y_{i,t+h}|x_{i,t}) = \alpha^{\tau}_{i} + \hat{\beta}^{\tau}_{x} * x_{i,t} + u_i, \tag{6}$$

where, for each quantile q and country i, α is a constant term, $\hat{\beta}^{q}_{x}$ is the estimated quantile coefficient associated with risk factor x at time t, and u_i is a country fixed effect. The GaR model programmed in the code that accompanies this article is adapted to be used with panel data, an improvement compared to previous models such as the IMF GaR model.

2.3 From quantile regressions to GDP growth distributions

The predicted values from the quantile regression are the main input to construct expected conditional distributions of GDP growth under specific methodologies. For instance, the quantiles could be chosen in equally spaced values, such as in $\{0.05, 0.10, \ldots, 0.95\}$, to allow for a more faithful representation of the support and the density of the distribution. It follows that one needs to estimate the entire distribution of expected GDP growth to obtain distributional quantities of interest, such as standard deviations of future growth.

To fit the entire distribution, one can follow parametric or non-parametric approaches. The former ones do not impose a particular shape for the distributions, simplifying the assumptions of the estimation. However, we do need to choose a kernel function and a bandwidth parameter. The kernel function assigns weights to the information contained within the bandwidth. Then, for a given bandwidth ω, the observations around the point x_i that are in the set $[x_i - \omega, x_i + \omega]$ are weighted according to the kernel function. For example, if one uses a constant kernel, each of the points within the described set will be equally weighed on the estimation of the fitted values.

Intuitively, the bandwidth represents how much information around a determined point is used to estimate the function around it. A very narrow bandwidth ($\omega \to 0$) implies using a series of delta functions around the observed points in the probability distribution function. Then, depending on the kernel, the cumulative distribution function will be approximated by a series of step functions. Conversely, the use of a very wide bandwidth ($\omega \to \infty$) implies using the kernel function on the whole collection of observations and, consequently, the probability density function will

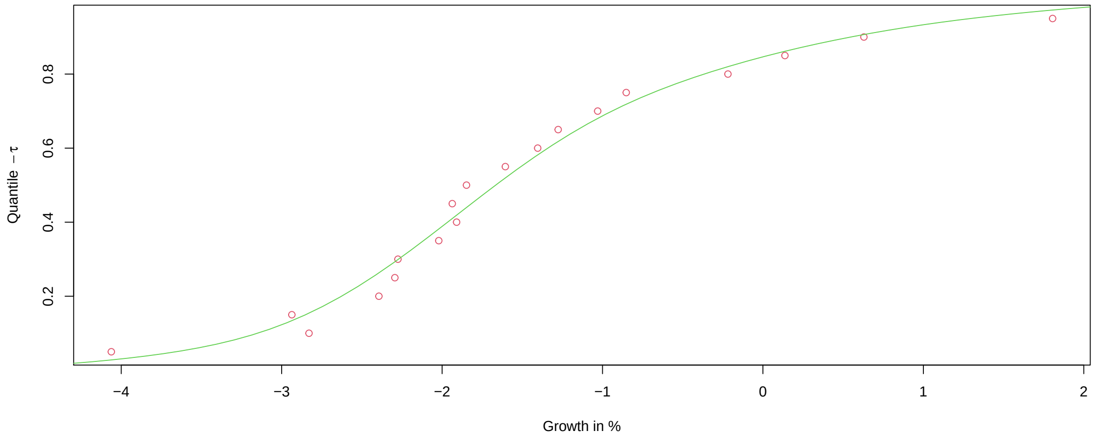


Figure 5: Representation of a set of estimated quantiles (red dots) and their associated fitted cumulative distribution using a non-parametric approach (green line).

be the kernel function centered on the mean of the sample. This latter approach represents a completely smoothed approximation that would not receive information about the sample points.

The specification implemented in the GaR model described below uses a tri-weighted kernel function, while the corresponding bandwidths are those suggested by Fan and Gijbels (1996). An interested reader could apply more sophisticated methods for choosing the bandwidth (see, e.g., Tsybakov, 2008, and Turlach, 1993). Figure 5 schematically represents some estimated quantiles (red dots) and their associated fitted distribution using the non-parametric approach described above. Note that such a figure must be estimated for each time period and for each country separately.

There are alternative parametric approaches used in the GaR literature, such as the one followed by Adrian et al. (2019), where one has to estimate the best t-skew distribution that fits the predicted quantiles. More specifically, one has to find the best parameters that minimize the sum of square differences between the predicted and fitted quantiles.[5] While each fitting procedure has its own pros and

[5] We note that the estimates of future growth distributions obtained from our model are similar

cons, our GaR model follows a non-parametric approach in consideration of the following features of parametric procedures:

- Parametric procedures are computationally more intensive, compared with non-parametric approaches. This is because they are based on 4-parameter functions that have to be estimated for each date and country.
- Moreover, such estimates are sensitive to the choice of the initial conditions, an aspect that matters specially under adverse macrofinancial scenarios.
- Finally, parametric approaches impose a distributional shape that could not be the adequate to represent the GDP growth distribution of individual countries.

The arguments above suggest that our non-parametric approach to fit the GDP growth distributions provides a more flexible and simpler method that ensures the replicability of the model in different contexts, avoiding large computational burdens. Despite having followed this approach, we have paid special attention in making the underlying GaR code flexible enough to incorporate alternative fitting procedures that can be added depending on each analyst's preferences.

2.4 Interpreting GaR results

Once the expected distribution of future GDP growth (conditional on macrofinancial conditions) has been estimated, the GaR method provides a flexible platform to explore the risk of negative GDP growth rates. The broad range of applications can be grouped into three main categories: macroeconomic risk assessment, scenario analysis, and policy evaluations. The most common output of a GaR model is the estimation of the probability of recession over time, conditional on the stance of macrofinancial conditions. That is, an analyst can explore different possible scenarios depending on the conditions identified as affecting GDP growth prospects.

regardless of the use of parametric or non-parametric fitting procedures.

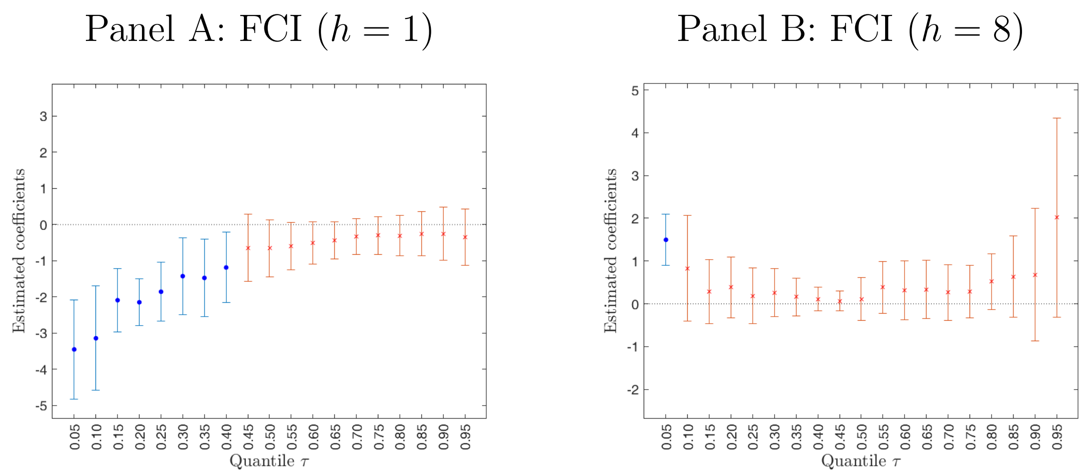


Figure 6: Results from a quantile regression. This panel reports an example of how results from a quantile regression can be visualized and interpreted. Panel A reports the coefficients with their corresponding 95 percent confidence intervals from a regression of GDP growth against a financial conditions index (FCI) with a one quarter horizon ($h = 1$). Panel B shows the results from the same estimation with an eight quarters horizon ($h = 8$). The x-axis represents the GDP growth quantiles corresponding to each coefficient, whose scale is depicted in the y-axis.

To interpret the results of a GaR model, a first step is to cautiously inspect the results from the QR model. For expositional purposes, Figure 6 depicts the results from a QR regression of GDP growth against a financial conditions index (FCI), which increases with the degree of financial stress in an economy. The estimation is based on a sample of five Latin American countries for the period from 1990 to 2020. The estimation depicted on Panel A shows that an increase in FCI (i.e., tighter macrofinancial conditions) can be associated with a decrease in the lower quantiles of the GDP growth distribution, one-quarter-ahead.[6]

This result implies that financial shocks will shift the GDP growth distribution to the left, leading to larger GDP growth losses. A one unit increase in FCI would reduce, for instance, the 0.3 quantile of GDP growth by approximately 1.3 percentage points. Given that this quantile was estimated at a GDP growth rate of approximately -2% in the distribution (see Figure 5), this increase in FCI would reduce the 0.3 quantile to -3.3%.

[6] These estimations are obtained from a panel including data from Brazil, Chile, Colombia, Mexico, and Peru.

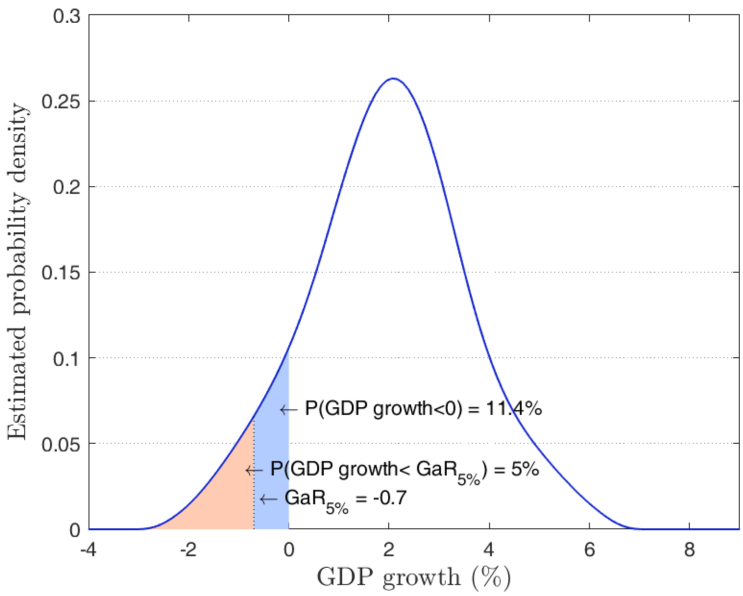
This interpretation will be sensible to the the horizon h chosen for the estimation. For example, it is likely that larger horizons will invert the sign of the identified relationship, capturing the fact that a recovery phase following a period of crisis will come along with increases in GDP growth. In fact, Panel B on Figure 6 shows that in a 8-quarter-ahead horizon, an increase in FCI is associated with increases in the lower quantiles of GDP growth.

Having estimated the conditional GDP growth distribution, the main result from the GaR analysis can be derived. An example of an estimated GDP growth distribution is depicted on Panel A in Figure 7. Given this distribution, we conclude that there is a 5 percent probability that real GDP growth will fall by at least 0.7 percent over the next quarter ($h = 1$). Using the GaR wording, this conclusion can be summarized as a GaR of -0.7 percent 1-quarter-ahead with a 5 percent probability. This distribution allows also to derive the probability of experiencing a negative GDP growth rate over the given horizon, which in this example corresponds to 11.4 percent.

The GaR estimation can be computed for each quarter over a long period using recursive rolling windows, illustrating ongoing and historical trends in the relationship between macrofinancial conditions and GDP growth. An example of this exercise is depicted on Panel B in Figure 7 for our sample of 5 Latin American countries. The GaR estimations capture well known events affecting these economies, such as the 2008 global financial crisis (GFC), and the COVID-19 pandemic. In these major events we observe large increases in the estimated GaR. For example, around the GFC the estimates went from a Growth at Risk of around 0 percent to approximately -10 percent.[7]

[7] The example depicted in Figure 7 can be extended to different metrics that can be drawn from the GDP growth distribution. Researchers can, for instance, replicate the figure on Panel B (GaR Estimates) by alternative measures. A few examples that have been used in the literature include the skewness of the distribution, its standard deviation computed using rolling windows, or the probability of negative GDP growth plotted over time.

Panel A: Expected GDP Growth



Panel B: GaR 1 quarter ahead

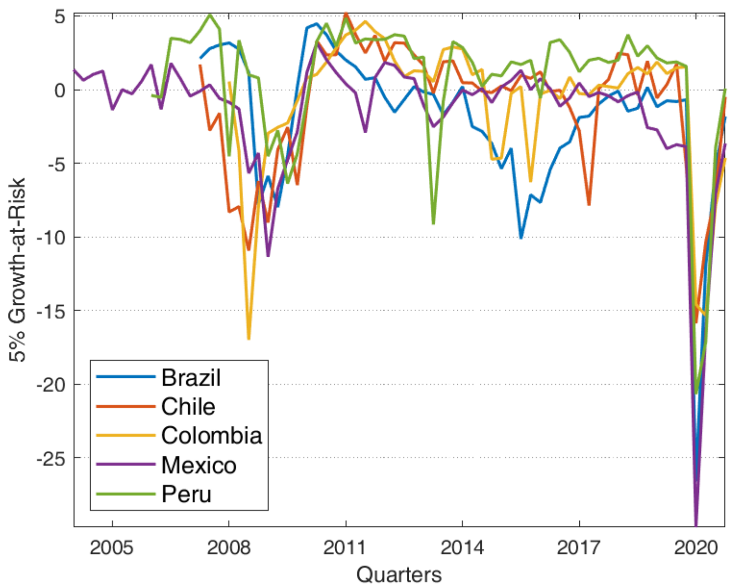


Figure 7: Results from a GaR estimation. This panel reports an example of results derived from a GaR model. Panel A shows the expected GDP growth distribution, conditional on the stance of macrofinancial conditions in a particular quarter. In this example the distribution shows that GDP growth will be at least -0.7 percent with a 5 percent probability one-quarter ahead. The probability of experiencing a negative GDP growth rate is estimated at 11.4 percent. Panel B depicts the GaR estimation for a set of Latin American economies estimated for each quarter between 1990 and 2020 using recursive rolling windows and for a 1-quarter-ahead horizon. These estimations are drawn from a panel including data from Brazil, Chile, Colombia, Mexico, and Peru.

Panel A: Predictive Score Panel B: Expected Shortfall and Longrise

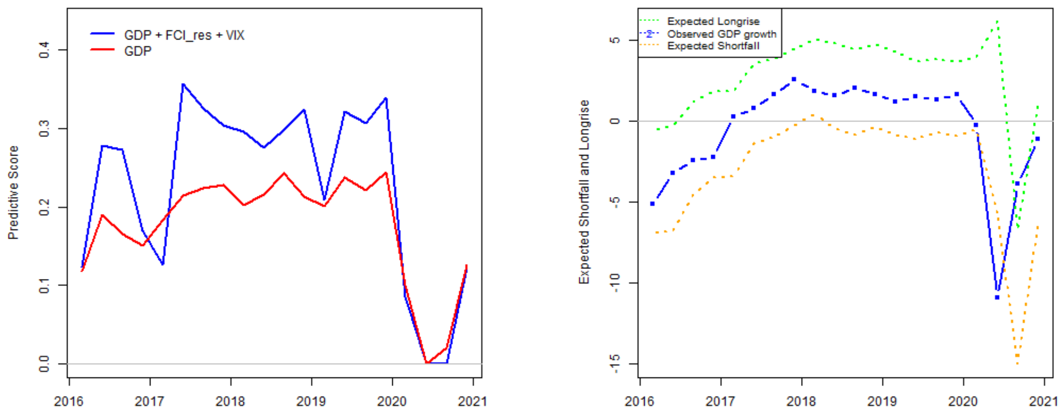


Figure 8: Post-estimation metrics for Brazil. This figure shows the predictive score, the expected shortfall, and the expected longrise, for Brazil from the panel data. On Panel A, we report the predictive score of the full model with lagged GDP growth, orthogonalized FCI, and VIX as regressors (blue), as well as a simpler model including only lagged GDP growth (red). On Panel B, we report the expected shortfall (orange line), the expected longrise (green line), and the observed GDP growth as reference (blue line).

An important aspect to consider when estimating GaR models is how to evaluate the precision and the economic relevance of the estimations. Following Adrian et al. (2019), we compute a set of post-estimation tests to shed light on the performance of the distribution estimation $\hat{f}_{t+h}$, represented by the fitted distribution obtained from quantiles $\hat{Q}^{\tau}_{i,t}$ as in Eq. 6.

First, we measure the precision of the estimates by using the predictive score PS_{t+h} of the estimation, given by:

$$PS_{t+h} = \hat{f}_{t+h}(\bar{y}), \tag{7}$$

where $\bar{y}$ is the observed value of the dependent variable at $t+h$, and PS_{t+h} corresponds to the likelihood of realized values under $\hat{f}_{t+h}$.

Second, we consider a measure of the tail risk given the estimated distribution, defined either as the shortfall function SF_{t+h} (left tail):

$$SF_{t+h} = \frac{1}{\pi} \int_0^\pi \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t) d\tau, \tag{8}$$

or as the longrise function LR_{t+h} (right tail), defined as:

$$LR_{t+h} = \frac{1}{\pi} \int_{1-\pi}^1 \hat{F}^{-1}_{y_{t+h}|x_t}(\tau|x_t) d\tau, \tag{9}$$

where $\hat{F}^{-1}$ is the inverse of the associated cumulative density function to $\hat{f}$, and π is a probability level. SF_{t+h} corresponds to the conditional mean of $\hat{f}$ on the left tail, that is, the expected value once we go below the quantile associated to π. Lower expected shortfalls represent heavier left tails and riskier distributions. Similarly, LR_{t+h} is the conditional mean to values above the quantile associated to $1 - \pi$.

In Figure 8, we illustrate these metrics using the estimation of the GaR model for Brazil for the period 2016-2020, and a time horizon of one quarter. In Panel A we show the results of the predictive score metric when comparing a baseline model including only lagged GDP growth as an explanatory variable (red line) against a more complete model including also the VIX index and the orthogonalized FCI index (blue line). In general, the more complete model has a higher predictive score.

Panel B shows the expected shortfall (orange line) and longrise (green line), capturing the tail risk of the estimated distributions for Brazil. As a reference, we include the observed GDP growth rate in blue. The upward trend towards 0 between 2016 and 2018 reflects a scenario in which the tail risks are reduced. On the contrary, the sharp decrease in the measure following the outbreak of the COVID-19 pandemic in 2020 implies an increase in the left-tail risk. Even more, in 2020 some observed values fell outside the respective range of the two metrics.[8]

[8] This has a stark contrast with the Great Financial Crisis, also captured in the panel. We note that prior to 2020, the parallel movement of the metrics on Panel B is due in part to a persistent shape in the estimated distributions, which only vary by a right shift during that period. This trend contrasts with the dynamics after the COVID-19 outbreak, where the realized GDP growth

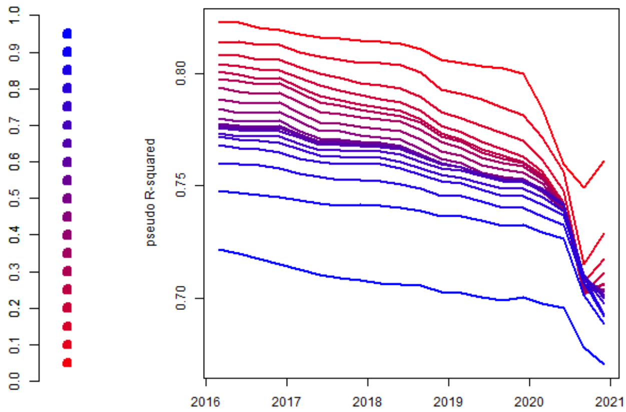


Figure 9: Pseudo R^2 of the panel estimation. The values reflect the explanatory capacity of the model $\hat{f}$ beyond $\hat{g}$. We mark the ordered quantiles in a shade from red to blue from lower to higher quantiles, according to the color scale on the left. Lower quantiles tend to have a better fit than higher quantiles, except for the period of the COVID-19 crisis.

Finally, researchers can also rely on a pseudo-R^2 metric computed for each quantile estimated on $\hat{f}$ and $\hat{g}$ in Eq. 9, as shown in Figure 9. In this figure, the color scale that goes from red to blue represents a range of lower to higher quantiles, respectively. In general, the model tends to have a better fit for the lower part of the distribution. However, the figure clearly reflects how the model loses its explanatory power around the COVID-19 crisis, especially for the lower quantiles.

Overall, the COVID-19 crisis in 2020 exhibits an atypical pattern in all three previous panels, representing a period in which the predictive score of the model decreases and the left-tail risk increases, while the fitting capacity of the model is deteriorated.

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rate falls outside the already shifted risk metrics as a consequence of this unexpected shock.

3 Financial Conditions Index

This section describes the construction of a Financial Conditions Index for a panel of Latin American economies using country-level data. This index follows and adapts the Country-Level Index of Financial Stress (CLIFS) methodology developed by Duprey et al. (2017). For the purposes of the index, we define financial stress as financial turbulence for several markets and asset classes. This index can be used to feed a GaR model with a proxy of domestic macrofinancial conditions.

The construction of the CLIFS is based on the indicator of systemic stress (CISS) proposed by Holló et al. (2012). The construction of the CISS exploits the correlation of financial stress across different market segments to create a composite indicator. These correlations are therefore used as weights to aggregate different sub-indices. By following this approach, the aggregated indicator considers the market-to-market spillovers that can be triggered when a single segment experiences a stress scenario.

The CLIFS relies on data from three financial market segments:

- Equity markets, as captured by a stock price index (STX);
- Bond markets, as captured by government yields for a fixed maturity (Ryr), preferably based on 10-year bonds; and
- Foreign exchange markets, as captured by the real effective exchange rate ($rEER$) computed as the geometric average of bilateral exchange rates weighted by bilateral trade volumes.

We use data from 2004 to 2020 for Chile, Mexico, Colombia, Peru and Brazil to compute the index. Most data are taken from Haver Analytics and from central banks' websites, providing information available at a daily and monthly frequency. The series of real effective exchange rates were obtained from the International Monetary Fund (IMF) at a monthly frequency. Despite our focus on a set of Latin

American economies, the statistical code generated to compute the CLIFS index can be extended to include other countries or variables of interest.

Three main reasons justify our choice of following the CLIFS methodology when constructing a financial stress index. First, this approach reduces concerns about the comparability of financial stress indicators across countries, as we rely on sub-indices based on simple variables which are commonly used in cross-country comparisons. Second, we can consider the contribution of financial stress sub-indices which do not necessarily co-move with each other. Finally, the sensitivity to outliers affecting alternative aggregation procedures such as PCA may worsen the problem of a sample with atypical values or stress events (Kremer et al., 2012).[9]

For each market segment, the CLIFS approach constructs two indicators reflecting each two characteristics of financial stress:

- Uncertainty, measured with a volatility metrix; and
- Market deterioration, measured with a cumulative performance metrix.

Given the long time series often used in GaR estimations, a relevant question is how a researcher can ensure the comparability of the stress scenarios over time. A first procedure consists in adjusting the data to purge the effects of inflation. To go from nominal stock market indies (STX) and government yields (Ryr) to real values we consider the following transformations:

- $rSTX_t$ is obtained by dividing STX_t by the corresponding interpolation of a daily consumer price index CPI_t.[10]
- $rRyr$ is obtained by removing the estimated annual inflation $\frac{CPI_t - CPI_{t-260}}{CPI_{t-260}}$ using interpolated CPI_t data values from Ryr. The time frame between $t-260$ and t stand for a year in workdays.

[9] We note that in the presence of outliers, the total variance computed in PCA estimations will be affected by these atypical observations, introducing a bias.

[10] Consumer price indices are produced on a monthly basis (see eq. 12, Appendix A.1).

- By construction, the measure of foreign exchange market stress $rEER$ is already expressed in real terms.

The uncertainty dimension is calculated using return-like variables as follows:

- The daily logarithmic return for equity, i.e., the logarithm of $rSTX_t$ divided by $rSTX_{t-1}$ and labeled as $lnSTX_t$.
- The daily changes as differences for bond markets, i.e., $rRyr_t$ minus $rRyr_{t-1}$, labeled as $chRyr$.[11]
- The monthly logarithmic return for the real exchange rate, i.e., the logarithm of $rEER_t$ divided by $rEER_{t-1}$, labeled as $lnEER_t$.

We further standardize these variables by scaling them with the standard deviation $\sigma_{X,t}$ of variable X considering the observations comprised in the past 10 years prior to time t. [12] This procedure ensures that the overlapping rolling windows generate comparable values. [13] In terms of notation, this procedure yields the following scaled variables:

- From $lnSTX$ to $ln\widetilde{S}TX$.
- From $chRyr$ to $ch\widetilde{R}yr$.
- From $lnEER$ to $ln\widetilde{E}ER$.

With these variables at hand, the stress indicators can be summarized as follows:

- Equity market stress indicator

[11] We use changes and not growth rates since, for some periods, very low real yields would create excessively large variations when measured as growth rates.

[12] For the first ten years of observations, we set the denominator constant and equal to the standard deviation of the first ten-year rolling window.

[13] Scaling by the standard deviation of the entire series would produce another series with the same pairwise relative magnitudes.

- $VSTX$: the monthly realized volatility, computed as the monthly average of absolute daily log-returns $ln\widetilde{STX}$.
- $CMAX$: the cumulative maximum loss, computed as the loss at each point in time compared to the highest level of the stock market index in a rolling two-years period.

- Bond market stress indicator
  - Vyr: the monthly realized volatility, computed as the monthly average of absolute daily changes in $ch\widetilde{R}yr$.
  - $CDIFF$: the difference between the maximum real government bond spread $rRyr$ (in basis points) with respect to United States bonds and the current spreat at time t in a two-years window.

- Foreign exchange market stress indicator
  - $VEER$: the realized volatility, computed as the absolute value of $(ln\widetilde{E}ER)$.
  - $CUMUL$: the cumulative change over six months in the real effective exchange rate, computed as the absolute value of the difference in a semester $|ln\widetilde{E}ER_t - ln\widetilde{E}ER_{t-6}|$, reflecting that longer-lasting changes in the real effective exchange rate should be associated with more severe stress.

The next step is to aggregate these variables into a single indicator. Given the different scales in which the indicators are computed, we begin by transforming each variable Z into $\hat{Z}$, corresponding to a map of its values into the $[0, 1]$ interval. This standardization can be achieved with two alternative methods:

- Method A: The transformation is carried out by the evaluation of the empirical cumulative density function obtained from an expanding time window. For the observation z_t, we map it into $\hat{z} = F_{Z,T}(z_T) = \frac{k}{T}$, such that z_T is the k-th order statistic from the set of observed values $\{z_1, \ldots, z_T\}$. For all the times T that fall below an initial threshold $T \leq T^\star$, we set $F_{Z,T} = F_{Z,T^\star}$.

- Method B: The transformation is done by a successive escalation by a maximum value drawn from within an expanding time window. For the observation z_t, we map it into $\hat{z} = S_T(z_T) = \frac{z_T}{z^\star}$, where $z^\star = \max_{t\in\{1,\ldots,T\}} z_t$, that is, the maximum value from the set of observed values $\{z_1, \ldots, z_T\}$. For all the times t that fall below an initial threshold $T \leq T^\star$, we set $S_T = S_{T^\star}$.[14]

By selecting the Method A, one gives more weight to *ordinal* relations among observations, while Method B is closer to a re-scaling of the original series giving more importance to *relative magnitudes*.

As an example, Figure 10 depicts the series of the cumulative difference stress indicator $CDIFF$ from 2007 to 2020 for Chile (blue lines), contrasted with standardized series from Methods A and B (orange lines). In the left panel, we mark the points in time t_1 and t_2, representing the local peaks in the original series. We note that despite the difference in magnitude, Method A reduces the difference between the peaks more than proportionally in the standardized values, since t_2 is closer in order to t_1. Given that the value at t_1 is the overall maximum and occurs earlier in time, the standardization by Method B produces a scaled version of the original series, explaining the overlap in the panel (despite the different scales).

[14] This method is not included in the approach by Duprey et al. (2017)

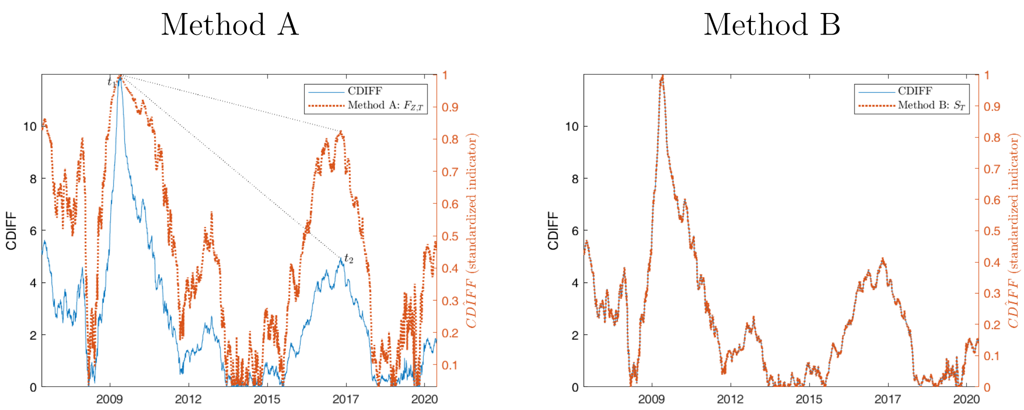


Figure 10: Standardization of the stress indicators $CDIFF$**.** This figure illustrates the standardization Methods A and B using as an example the GaR estimations for Chile between 2007 and 2020. The left panel shows a standardization with Method A, by the empirical cumulative density function. Method B is shown on the right panel, with a standardization based on maximum values obtained from rolling windows.

The transformed indicators are averaged within each market segment into the sub-indices $I_{STX} = \frac{1}{2}(V\hat{STX} + C\hat{MAX})$, $I_{Ryr} = \frac{1}{2}(V\hat{Ryr} + C\hat{MIN})$, and $I_{EER} = \frac{1}{2}(V\hat{EER} + CU\hat{MUL})$. That is, each sub-index has an equally weighted contribution from a volatility and a performative aspect. This transformation avoids imposing an arbitrary weight between the two sources of stress (i.e., the volatility and performance indicators).

As an example, consider Figure 11, in which we depict a toy example for the stock market index with four 20-day time windows. This example is intended to ease the interpretation of how the stress indicators react to changes in the underlying variable. Consider the first window as a base case, representing a scenario with loose financial conditions (despite the level of the variable). The next two windows depict a tightening in only one of the two stress indicators; its final impact will depend on their relative magnitudes. Finally, the fourth window shows the worst scenario with a deterioration in both stress indicators and an evident tightening in financial conditions. The sub-index reflects at its extreme values of 0 and 1 (right y-axis) stable growth versus an erratic and negative drift, respectively.

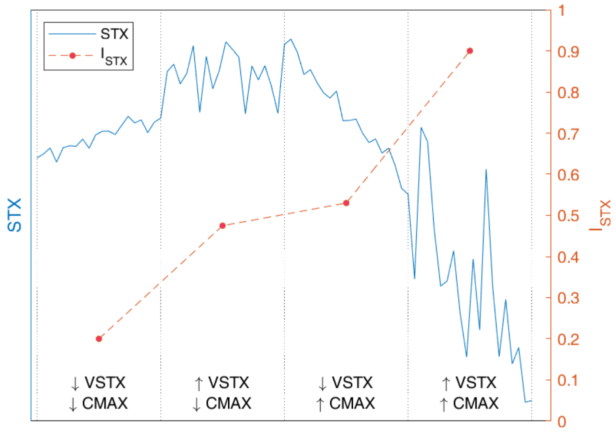


Figure 11: Toy example of STX. This figure shows a hypothetical example of the behavior of the stock market indicator. The blue line represents a stock index over four months, i.e., four 20-working day time windows separated by vertical black lines (which show the respective end of period). The dashed orange line represents the sub-index I_{STX}. For each time window, we describe the level of the stress indicators with either a high (↑) or low (↓) mark at the bottom of the plot.

Based on the series of sub-indices, we next produce the series of pairwise correlations $\rho_{STX,Ryr}$, ρ_{STX_EER} and $\rho_{Ryr,EER}$, estimated with an exponentially weighted moving average.[15] Finally, the aggregation of the sub-indices into the Financial Conditions Index is represented by Eq. 10:

$$
\begin{aligned}
FCI_t \;=\;& I^2_{STX} + \rho_{STX,Ryr,t} I_{STX} I_{Ryr} + \rho_{STX,EER,t} I_{STX} I_{EER} \\
& + \rho_{STX,Ryr,t} I_{STX} I_{Ryr} + I^2_{Ryr} + \rho_{Ryr,EER,t} I_{Ryr} I_{EER} \\
& + \rho_{STX,EER,t} I_{STX} I_{Ryr} + \rho_{Ryr,EER,t} I_{Ryr} I_{EER} + I^2_{EER}.
\end{aligned} \tag{10}
$$

As Eq. 10 suggests, the calculation stems from a double matricial product between a vector index and the pairwise correlation matrix[16]. The complete expansion helps to highlight the spillover effects between segments as one of the potential sources of financial markets stress in the aggregate index. The main takeaway of Eq. 10 is that financial stress will arise either (i) due to an (idiosyncratic) increase

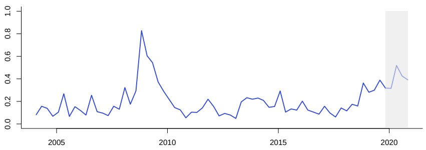
[15] This procedure is a simplification of a GARCH estimation. We refer to Duprey et al. (2017) for further details.

[16] The vectorial and matricial notation is described in detail in the appendix

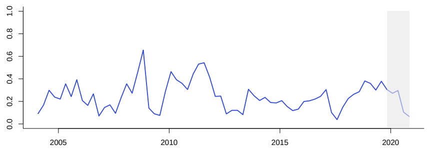
Box 1. Estimating the Financial Conditions Index for Mexico

This Box describes the performance of the Financial Conditions Index and its components for Mexico during the period from 2004 to 2020. The COVID-19 pandemic is highlighted by a shaded area in the graphs below.

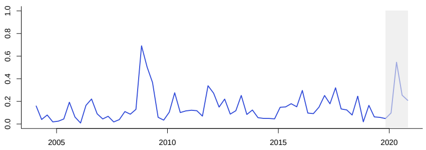
A tightening in the financial conditions is shown as an increase in the value of the composite index and its underlying sub-indices. At the onset of the COVID-19 crisis, the index signals a deterioration, particularly in the first quarter of 2020. In the same time window, the individual sub-indices tend to peak, while the correlations tend to align closer to 1.



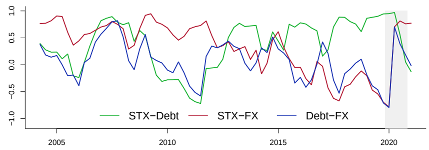
(a) Equity market sub-index I_{STX}



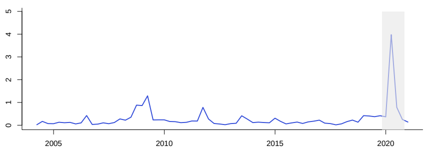
(b) Bond market sub-index I_{Ryr}



(c) Foreign Exchange sub-index I_{EER}



(d) Pairwise cross-correlations



(e) FCI for Mexico

Overall, the Mexican example shows that the sub-indices can capture relevant shocks in the Mexican economy, such as the global financial crisis or the COVID-19 pandemic. Also, the high levels of the bilateral correlations at the onset of these major shocks illustrate the amplification effect in the economy represented by the spillovers between market segments.

in the sub-indices, (ii) due to an increase in the pairwise correlation between market segments, or (iii) due to a combination of (i) and (ii).[17]

To illustrate the estimation of the financial conditions index and the interpretation outlined above, Box 1 discusses for the case of Mexico the performance of the different sub-indices (Panels a, b, and c) and the aggregate index (Panel e) over the 2004-2020 sample period with a quarterly frequency. This Box also shows the performance of the bilateral cross-correlations between sub-indices used in the aggregation procedure (Panel d).

The year 2020 is shaded to highlight the period in which the effect of the COVID-19 crisis can be visualized. Moreover, Panel d shows how the bilateral correlations between sub-indices vary over time, highlighting the importance of capturing the time-varying nature of market-to-market spillovers that explain the aggregate implications of segment-specifics increases in market stress.

4 Applying GaR for policy analysis

This section summarizes a few alternatives for policymakers when applying GaR methods for policy analysis. While a detailed description of these approaches is beyond the scope of this article, the explanation below aims at guiding an interested reader in identifying possibilities for policy analysis within the GaR framework. We refer to the IMF report by Prasad et al. (2019) as a reference on more detailed descriptions of possible policy applications.

GaR as a benchmark to compare GDP growth forecasts. One application considers using the estimated GDP growth distribution as a benchmark to compare alternative GDP growth forecasts. This comparison can take two possible forms.

[17] We note that Eq. 10 does not include any country-specific parameters. This feature of the index facilitates the comparability across countries by remaining agnostic about the actual contribution of different market segments to financial stress.

First, an analyst can evaluate whether an alternative forecast of the GDP growth distribution is similar in terms its related GaR at 5 percent of probability compared to the baseline GaR model. More optimistic or pessimistic forecasts would identify a different mode and shape in the GDP growth distribution. Second, the estimated GaR model can be used to explore whether alternative forecasts provide a reasonable assessment of macrofinancial risks identified in the quantile regressions. Prasad et al. (2019) provide a few examples of how these comparisons can be implemented.

Scenario analyses under different macrofinancial conditions. GaR estimates can be used to perform scenario analysis in which relevant macrofinancial risks are assumed to be shocked. Graphically, this scenario analysis resembles the exercise reported in Figure 2. Since the GDP growth distribution is estimated conditional on the stance of macrofinancial risk, an analyst can ask how the distribution would change given tighter macrofinancial conditions. The relevant output measure in this case is the increase in GaR given a one standard deviation increase in, say, a financial conditions index.[18]

Identifying changing patterns in GDP growth risk factors. GaR models can also be used to trace the relevant importance of different macrofinancial variables in affecting GDP growth prospects. The relevance of risk factors can differ both across countries as well as over time within a given country. The IMF has used GaR models to identify, for instance, that downside risks to future growth can be disproportionately affected in certain countries by factors such as commodity prices, China's growth, or trends in global US Dollar prices. The size of the risk signal represented by these variables is likely to change even in short-term horizons.

[18] It should be recalled that the variables included in the quantile regressions may affect the distribution in different ways, leading either to a shift in the distribution's mean or the changes in the shape of the distribution's tails. This analysis can tell us, for instance, that a one standard deviation increase in a particular macrofinancial risks (e.g., the VIX index) increases the probability of negative growth by a given percent. Adrian et al. (2019) provides several examples of how these scenario analyses can be performed.

Box 2. Example of a policy evaluation using quantile regressions with interaction terms

The baseline QR model represented in Eq. 2 can be adapted to incorporate interaction terms between macrofinancial variables of interest and variables capturing the stance of policy actions. Some studies have relied, for instance, on data from the international macroprudential databases by Cerutti et al. (2017) or Nier et al. (2018).

Eq. 11 shows an example of an interaction QR model. In this example the variable of interest is the financial conditions index FCI which is computed for country i at quarter t for a particular horizon of analysis h:

$$\begin{aligned} Q_{GDP_{i,t+h}} = \alpha_h(\tau) + \beta_{1,h}(\tau)FCI_t + \beta_{2,h}(\tau)Policy_{i,t} \\ + \beta_{3,h}(\tau)FCI_t * Policy_{i,t} + \varepsilon_{i,t} \end{aligned} \tag{11}$$

This model includes an interaction term between FCI and the variable $Policy$, which represents a macroprudential policy index ranging from 0 to 1 depending on the tightness of macroprudential regulation in country i during quarter t. Double-causality concerns can be addressed, for instance, by lagging the variable $Policy$ in further quarters (e.g., t-3) in order to separate de policy decisions from current macro trends.

The coefficient $\beta_{3,h}(\tau)$ represents the heterogeneous effect of FCI on a given quantile τ of the GDP growth distribution conditional on a particular stance of the policy index. For example, Eguren-Martin et al. (2021) use a similar model to find that the effect of tighter global financial conditions on the distribution of capital flows is moderated when macroprudential policies are in place.

Therefore, frequent updates of GaR estimates can guide policymakers in timely identifying sectoral risk exposures.

Policy evaluation and macroprudential regulation. Analysts interested in evaluating the effect of policies on the expected GDP growth distribution may also adapt the GaR environment for this purpose. Relevant questions include, for instance, whether differential shifts in the distribution occur when macroprudential policies are in place. Similarly, one could exploit panel settings to explore differences in the effect of global financial conditions depending on countries' stance of monetary, fiscal, or regulatory policies. Overall, three approaches for these analyses can be considered.

- **Heterogeneous effects across periods or jurisdictions.** The simplest approach to explore the effect of policies in a GaR environment is to repeat the baseline estimation considering separately periods during which certain policies have been implemented. Similarly, one can group countries/regions according to shared characteristics in terms of policy actions (e.g., those with active counter-cyclical capital buffers).

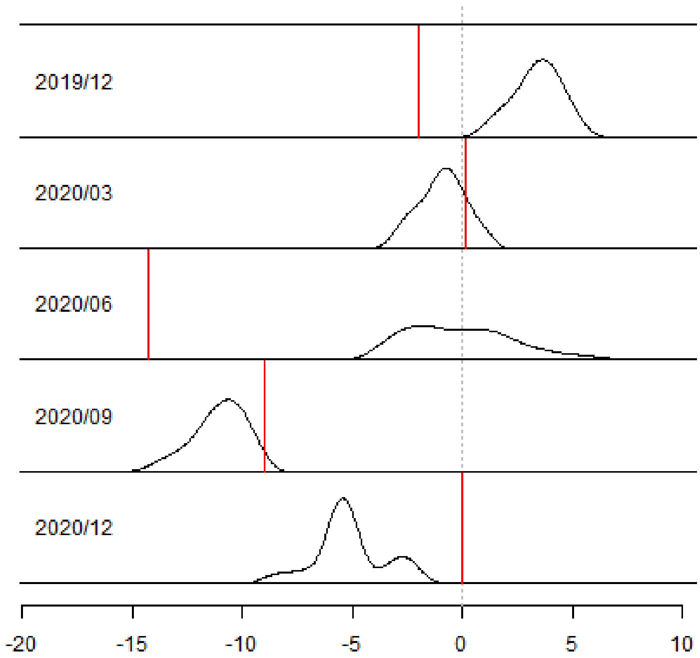
- **Including macrofinancial variables.** Alternatively, one can include a proxy for policy measures as a further variable alongside the other macrofinancial variables in the quantile regressions. For example, one can use country-level proxies for the intensity of the use of macroprudential policies. This variable would enter the GaR model as a stand-alone variable, allowing for exploring changes in GaR driven by the activation of policy measures.[19]

- **Interaction models.** A more complex but insightful approach is to adapt the baseline GaR model to include interaction terms between macrofinancial variables and proxies for the stance of relevant policies in the quantile regressions.

[19] Galán (2020) provides an example of this approach, finding that macroprudential policies do affect the left tail of estimated GDP growth distributions, with heterogeneous effects depending on the horizons considered.

Box 3. The performance of GaR estimation under unexpected exogenous shocks

The example of Chile helps to illustrate the performance of a GaR estimation when an economy faces an unexpected an exogenous shock. The chart below depicts the estimated distributions for the Chilean case in black, and the actually observed GDP growth rates in red for the period from 2019Q4 to 2020Q4. In this period, Chile experienced first a shock represented by large protests and social unrest in 2019Q4. Then, in 2020Q1 the country was further affected by the COVID-19 pandemic.



When these shocks hit the economy, the observed GDP growth rate falls to the left of the expected distribution, while sharp recovery trends such as the one observed in 2020Q4 also fall outside the range of the estimated distribution. This exercise reflects that GaR models are estimated by a backward-looking approach that may not anticipate sudden changes in scenarios not captured by macrofinancial fundamentals.

This approach, for which we provide an example in Box 2, allows researchers to explore non-linear effects of macrofinancial variables on the GDP growth distribution.[20]

5 Conclusion

This article provides a technical description on estimating GaR models using an R-based statistical code developed at the Center for Latin American Monetary Studies (CEMLA) under the frame of the C-GARP initiative. The C-GARP initiative provides an open-source platform for GaR estimations, allowing users to implement analyses tailored to the needs and characteristics of different jurisdictions.

This statistical code provides a platform to perform four key steps in the GaR approach: (i) the construction of a country-level financial stress index based in the CLIFS methodology; (ii) the estimation of quantile regressions exploring the effect of macrofinancial conditions on GDP growth; (iii) the computation of fitted expected GDP growth distributions; and (iv) the evaluation of stress scenarios.

The exercises reported for a set of Latin American economies show that the GaR model can provide a useful assessment of how the severity of systemic risk impacts an economy. While the conclusions drawn from a GaR model can be informative to calibrate macroprudential policy responses, caution must be exercised when evaluating policy trade offs and ultimate causal sources of financial stress. We therefore remark that incorporating GaR models for policy analysis is best done when different methodological tools are combined within a more general framework. Overall, our conclusions support the notion that GaR models can fill missing gaps when quantifying the severity of macrofinancial risks.

[20] Eguren-Martin et al. (2021) provide an example of this approach by estimating the conditional distribution of capital flows in a panel of emerging countries. The authors find that active macroprudential policies moderate the pass-through of macrofinancial shocks to shifts in the distribution of capital flows.

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A Appendix

A.1 FCI formulas

In this section we describe in more detail the process to obtain the Financial Conditions Index. The overall process is described in Figure 12, where the notation follows Duprey et al. (2017) and the variable names in the code, which are also summarized in Appendix A.2.

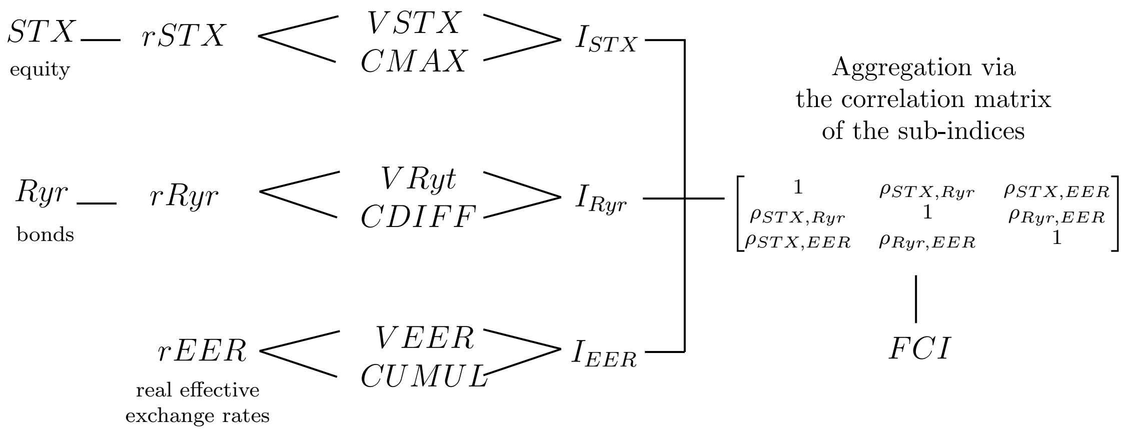

Figure 12: FCI computation process.

First, the following equation blocks specify the transformation process to construct the stress indicators for the equity, bond market, and exchange market segments mentioned in Section 3.

$$
\text{Equity market} = \begin{cases} rSTX_t = \dfrac{STX_t}{CPI_t} & (12) \\ lnSTX_t = \log rSTX_t - \log rSTX_{t-1} & (13) \\ ln\widetilde{STX}_t = \dfrac{lnSTX_t}{\sigma_{lnSTX_{t,t-lagTilde}}} & (14) \\ VSTX_t = \dfrac{\sum_{i=0}^{lagPeriod} \left| ln\widetilde{STX}_{t-i} \right|}{lagPeriod} & (15) \\ CMAX_t = 1 - \dfrac{rSTX_t}{\max_{i=0}^{lagWindow} ln\widetilde{STX}_{t-i}} & (16) \end{cases}
$$

$$
\text{Bond Market} = \begin{cases} rRyr_t = Ryr_t - \dfrac{CPI_t - CPI_{t-lagYear}}{CPI_{t-lagYear}} & (17) \\ chRyr_t = rRyr_t - rRyr_{t-1} & (18) \\ ch\widetilde{Ryr}_t = \dfrac{chRyr_t}{\sigma_{chRyr_{t,t-lagTilde}}} & (19) \\ VRyr_t = \dfrac{\sum_{i=0}^{lagPeriod} \left| ch\widetilde{Ryr}_{t-i} \right|}{lagPeriod} & (20) \\ CDIFF_t = rRyr_t - rRbs_t - \min\limits_{i=0}^{lagWindow} (rRyr_{t-i} - rRbs_{t-i}) & (21) \end{cases}
$$

The frequency of the information for the equity (STX) and bond markets (Ryr) is daily, so that t stands for a workday indexation. The consumer price index (CPI) is interpolated between available dates to estimate daily values. The rolling time windows parameters $lagTilde$, $lagPeriod$ and $lagWindow$ are set to resemble 10 years, one month, and 2 years, respectively.

When computing the variable $CDIFF_t$, the variable $rRbs$ represents the yield of an advanced economy used as a benchmark. In our case, we employ the U.S. economy.

$$
\text{Foreign exchange market} = \begin{cases} lnEER_t = \log(rEER_t) - \log(rEER_{t-1}) & (22) \\ \widetilde{lnEER}_t = \dfrac{lnEER_t}{\sigma_{lnEER_{t,t-lagTildeMonth}}} & (23) \\ VEER_t = \left|\widetilde{lnEER}_t\right| & (24) \\ CUMUL_t = \left|rEER_t - rEER_{t-lagWindowMonth}\right| & (25) \end{cases}
$$

The frequency of the foreign exchange market is monthly, so t stands for an end-of-month indexation. The rolling time windows parameters $lagTildeMonth$ and $lagWindowMonth$ are set to resemble 10 years and 6 months, respectively.

Second, we obtain the stress sub-indices from the stress indicators. We consider end-of-month $\hat{z}_t$ values for the daily stress indicators as our monthly values. Each stress indicator is transformed into the interval $[0, 1]$ according to one of the following methods:

- Method A:

$$
\hat{z} = F_{Z,T}(z_T) = \frac{k}{T}, \tag{26}
$$

  such that z_T is the k-th order statistic from the set of observed values $\{z_1, \ldots, z_T\}$. For all the times T that fall below an initial threshold $T \leq T^\star$, we set $F_{Z,T} = F_{Z,T^\star}$.

- Method B:

$$
\hat{z} = S_T(z_T) = \frac{z_T}{\max_{t\in\{1,\ldots,T\}} z_t} \tag{27}
$$

  For all the times T that fall below an initial threshold $T \leq T^\star$, we set $S_T = S_{T^\star}$.

for z_k in $\{VSTX, CMAX, VRyr, CDIFF, VEER, CUMUL\}$.

With the transformed stress indicators, we obtain the averages of the stress indicators for each segment, grouping them into a sub-index vector I_t:

$$\text{Sub-indices} = \begin{cases} I_{STX} = \dfrac{V\hat{STX} + CM\hat{A}X}{2} & (28) \\ I_{Ryr} = \dfrac{V\hat{R}yr + CD\hat{I}FF}{2} & (29) \\ I_{EER} = \dfrac{V\hat{E}ER + CU\hat{M}UL}{2} & (30) \\ I_t = [I_{STX}, I_{Ryr}, I_{EER}] & (31) \end{cases}$$

The final step consists in the aggregation of the sub-indices in a final stress index. We estimate the pairwise correlations for the elements in I_t with the following simplification:

$$\text{Estimation of variances and correlations} = \begin{cases} \sigma_{i,j,t} = \lambda\sigma^2_{i,j,t-1} + (1-\lambda)\bar{s}_{i,t}\bar{s}_{j,t} & (32) \\ \sigma^2_{i,t} = \lambda\sigma^2_{i,t-1} + (1-\lambda)\bar{s}^2_{i,t} & (33) \\ \rho_{i,j,t} = \dfrac{\sigma_{i,j,t}}{\sigma_{i,t}\sigma j, t} & (34) \end{cases}$$

Following Duprey et al. (2017), λ is set as 0.85.

With these correlations, we construct the correlation matrix:

$$C_t = \begin{bmatrix} 1 & \rho_{STX,Ryr,t} & \rho_{STX,EER,t} \\ \rho_{STX,Ryr,t} & 1 & \rho_{Ryr,EER,t} \\ \rho_{STX,EER,t} & \rho_{Ryr,EER,t} & 1 \end{bmatrix} \tag{35}$$

Finally, the Financial Conditions Index is the double product between the correlation matrix, and the sub-indices vector I_t:

$$FCI_t = I_t \cdot C_t \cdot I'_t \tag{36}$$

A.2 Variable Definition

Table 2: Variables definition

Variable	Definition	Unit and Sources
GaR Variables		
GDP Growth (GDP)	Year-to-year GDP growth rate.	Percent. (Haver Analytics)
VIX (VIX)	CBOE Volatility Index	Index unit. (CBOE)
FCI (FCI)	Financial Conditions Index.	Index unit. (Own calculation)
Financial condition index variables		
Capital (STX)	Stock-market index per country.	Index unit. (Investing.com, Yahoo Finance, CBs)
Government yield (Ryr)	Rate associated to a fixed node of a government bond.	100 bps. (Investing.com, CBs)
Real effective exchange rate ($rEER$)	The value of a country's exchange rate against a weighted average of several foreign currencies divided by a price deflator or index of costs.	Index. (IMF, BCRP)
Consumer price index (CPI)	Index of the average change over time in the prices of a market basket of consumer goods and services.	Index. (St. Louis FRED, Haver Analytics.)

Notes: This table provides a description of the main variables used for the empirical analysis reported in the paper. Sources are reported in parentheses. Sources are not reported for variables resulting from the transformation of other variables.

Table 2: Variables definition (continued)

Variable	Definition	Unit and Sources
Financial condition index variables		
Reference government yield (Rbs)	Government rate at a fix node of a reference advanced economy.	100 bps. (U.S. Treasury)
Reference consumer price index ($CPIbs$)	Consumer price index of a reference advanced economy.	Index. (St. Louis FRED.)
Real stock index ($rSTX$)	Deflated stock indices.	Scaled index.
Stock log-returns ($lnSTX$)	Daily log-returns of deflated capital market indices.	Percent.
Standardized stock log-return ($ln\widetilde{STX}$)	Standardization of log-returns with respect the s.d. over a rolling time window.	Percent.
Realized volatility of stock ($VSTX$)	Monthly realized volatility of the scaled stock indices log-returns.	Dimensionless.
Range-based stock stress ($CMAX$)	Comparison of the observed log-returns and the maximum over a rolling time window.	Percent.
Real yield ($rRyr$)	Real government yields	100 bps
Yield change ($chRyr$)	Daily yield changes	100 bps
Standardized yield change ($ch\widetilde{R}yr$)	Standardization of yields with respect the s.d. over a rolling time window.	Dimensionless.

Notes: This table provides a description of the main variables used for the empirical analysis reported in the paper. Sources are reported in parentheses. Sources are not reported for variables resulting from the transformation of other variables.

Table 2: Variables definition (continued)

Variable	Definition	Unit and Sources
Financial condition index variables		
Realized volatility of yield ($VRyr$)	Monthly realized volatility of the scaled yields.	Dimensionless.
Range-based yield stress ($CDIFF$)	Comparison of the observed yield gaps to the reference real yield and the its minimum over a rolling time window.	Percent.
Real exchange effective rate log-return ($lnEER$)	Montly log-return of real effective exchange rates.	Percent.
Standardized real effective exchange rate log-return ($\widetilde{EER}$)	Standardization of the log-returns with respect the s.d. over a rolling time window.	Dimensionless.
Realized volatility of real eff. ex. rate ($VEER$)	The absolute value of the observed log-return for the month.	Dimensionless.
Cumulative change ($CUMUL$)	6-month cumulative change.	Percent.
Transformed volatility-based stock stress ($V\widetilde{STX}$)	Transformed volatility-based stock stress.	Dimensionless.
Transformed volatility-based yield stress ($V\widetilde{Ryr}$)	Transformed volatility-based yield stress.	Dimensionless.
Transformed volatility-based ex. rate stress ($V\widetilde{EER}$)	Transformed volatility-based ex. rate stress.	Dimensionless.

Notes: This table provides a description of the main variables used for the empirical analysis reported in the paper. Sources are reported in parentheses. Sources are not reported for variables resulting from the transformation of other variables.

Table 2: Variables definition (continued)

Variable	Definition	Unit and Sources
Financial condition index variables		
Transformed range-based stock stress ($\widetilde{CMAX}$)	Transformed range-based stock stress.	Dimensionless.
Transformed range-based yield stress ($\widetilde{CDIFF}$)	Transformed range-based yield stress.	Dimensionless.
Transformed range-based ex. rate stress ($\widetilde{CUMUL}$)	Transformed range-based ex. rate stress.	Dimensionless.
Stock sub-index (I_{STX})	Average of the stock stress indicators.	Dimensionless.
Rate sub-index (I_{Ryr})	Average of the yield stress indicators.	Dimensionless.
Exchange rate sub-index (I_{EER})	Average of the exchange rate stress indicators.	Dimensionless.
Stock-rate correlation ($\rho_{STX,Ryr}$)	Estimation of the Stock-rate correlation.	Dimensionless.
Stock-ex. rate correlation ($\rho_{STX,EER}$)	Estimation of the Stock-ex. rate correlation.	Dimensionless.
Rate-ex. rate correlation ($\rho_{Ryr,EER}$)	Estimation of the Rate-ex. rate correlation.	Dimensionless.
Financial Conditions Index (FCI)	Financial Conditions Index, based on Country-Level Index for Financial Stress.	Dimensionless.

Notes: This table provides a description of the main variables used for the empirical analysis reported in the paper. Sources are reported in parentheses. Sources are not reported for variables resulting from the transformation of other variables.